

# Localized Kaluza-Klein Graviton and Cosmological Constant

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## Abstract

We study linearized graviton in the presence of a four-dimensional cosmological constant in two brane models with a warped extra dimension. In explicit models including bulk scalar fields, we calculate the masses of Kaluza-Klein modes of graviton and their interactions with matter on the visible brane. It is shown that the effects of the cosmological constant contribute by the equivalent size to the warp factor, masses and couplings and that bulk scalar fields can increase the effects. This is examined further independently of the forms of scalar potentials. Then it is found how the masses and couplings are described in terms of the warp factors and generic scalar potentials. A possibility that the masses and couplings are significantly changed by cosmological constant effects is discussed.

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# 1 Introduction

There has been much interest in phenomenological possibilities of higher-dimensional models with a warped extra dimension since it was shown that they may provide a solution to the gauge hierarchy problem. In the Randall-Sundrum model [1], the first Kaluza-Klein (KK) mode of graviton can have the mass of the order  $\mathcal{O}(1)$  TeV and the coupling with matter on the visible brane is of the order  $\mathcal{O}(1)$   $\text{TeV}^{-1}$ . This induces new effects which in principle can be seen at future colliders. The first KK graviton will be directly searched in the resonance production such as Drell-Yan process [2] or electron-positron pair annihilation [3]. Models with a warped extra dimension extensively has been studied also in phenomenological context including brane localized curvature, radion, bulk gauge bosons, bulk fermions, neutrino oscillations, grand unification, and supersymmetry. The progress has led to various possible ways of detecting an extra dimension. It however has been assumed that in most of these models the fine-tuning between the bulk curvature and brane tension makes the four-dimensional cosmological constant vanish. Since recent observations have supported that the cosmological constant is nonzero [4], it is interesting to examine new phenomenological possibilities at TeV scale by taking in this effect.

In the nonzero cosmological constant case, brane picture including localizability and stability has been inspected. Brane configurations with a cosmological constant are solutions to the setup in the Randall-Sundrum model, where the fine-tuning is relaxed [5]. If bulk scalar fields are taken into account, it also may be possible to address a explanation to the smallness of the cosmological constant with less tuning [6–9]. For these solutions, it has been found that localization of fields [10] and Newton’s potential [11] are analogous to those of the zero cosmological constant case [1, 12, 13] and that radion with a stable mass spectrum seems to need bulk scalar fields [14]. Since bulk scalar fields change background geometry, it is likely that they play a role to change phenomenological consequences.

In this paper, we calculate the masses and interactions of KK graviton on the visible brane in models including a warped extra dimension, a four-dimensional cosmological constant and bulk scalar fields. We first perform the analyses in explicit models where the warp factors are expanded into power series of the cosmological constant. One of the models is the pure gravity case and the other is the case of the scalar potential given in Ref. [7]. From the analyses at the first order of the power expansion, it is found that the masses and couplings receive more largely the effects of the cosmological constant in the model with the bulk scalar fields. This is understood from that the bulk scalar field increases the cosmological constant effect on the warp factor. Beyond the power expansion, it is required to examine whether bulk scalar fields enhance the cosmological constant effects. We analyze mode eigenfunction of graviton for generic warp factors without relying on the power expansion and special forms of scalar potentials. It is found how the masses and couplings are generally described by the warp factors and scalar potentials. Then it is shown that masses and couplings significantly can be different from those of the zero cosmological constant case.

This paper is organized as follows. In Section 2, we briefly review the formulation of KK modes of graviton in models with a warped extra dimension and a four-dimensional cosmological constant. Section 3 are devoted to analyzing masses and interactions in explicit

models. A relation of the warp factors with the masses and couplings is found. In Section 4, we derive a more general relation among warp factors, masses, and couplings. We summarize our results in Section 5.

## 2 Formulation

The model is formulated in the five-dimensional general relativity on the orbifold  $S^1/Z_2$  with the compactification radius  $R$ . The action of the model is

$$S = \frac{1}{2\kappa^2} \int d^4x \int dy \sqrt{-\det g_{MN}} \left( \mathcal{R} - \partial_M \Phi \partial^M \Phi - 2V(\Phi) \right) + S_{\text{source}} + S_{\text{matter}}, \quad (2.1)$$

with the five-dimensional gravitational coupling constant  $\kappa$ , the five-dimensional Ricci scalar  $\mathcal{R}$  and the scalar potential  $V$ . The four-dimensional and fifth space coordinates are denoted by  $x$  and  $y$ , respectively, and the capital Latin indices  $M, N, \dots$  label the five-dimensional indices. The action  $S_{\text{source}}$  is introduced as a source in order that two branes are consistently set at the fixed points of the orbifold. This action plays roles to parameterize the tensions of the branes and to stabilize the compactification radius. We simply assume that the source action controls the singularities along the direction perpendicular to the branes and that the cutoff energies on the branes are Planck scale at  $y = 0$  and TeV scale at  $y = \pi R$ . In the equation (2.1), the last term  $S_{\text{matter}}$  describes fields in the visible sector, which are confined to the brane at  $y = \pi R$ . The interactions of the bulk fields with these fields induce new effects in scattering processes at TeV scale.

We work with the scalar background independent of the coordinates parallel to the branes,  $\Phi = \Phi(y)$  and the line element

$$ds^2 = A(y)^2 (-dt^2 + e^{2\sqrt{\lambda}t} \delta_{ij} dx^i dx^j) + dy^2, \quad (2.2)$$

where  $\lambda$  indicates the four-dimensional cosmological constant. Although  $\lambda$  is not a parameter of the model, the smallness can be chosen by hand with the degrees of freedom of parameterizing the brane tension<sup>‡</sup>. The warp factor  $A$  necessarily involves  $\lambda$  dependently on the scalar background since the line element is the solution to the equations of motion derived from the action (2.1). In the equations of motion, the scalar potential can be composed of the superpotential-like function  $W(\Phi)$ ,

$$V(\Phi) = \frac{1}{8\gamma^2} \left( \frac{\partial W}{\partial \Phi} \right)^2 - \frac{1}{6} W^2, \quad (2.3)$$

where  $\gamma(y) = \sqrt{1 + 4\lambda/(WA)^2}$ . Then the other equations are written as the first order equations

$$\frac{A'}{A} = -\frac{1}{6} W\gamma, \quad \Phi' = \frac{1}{2\gamma} \frac{\partial W}{\partial \Phi}, \quad (2.4)$$

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<sup>‡</sup>The warp factor may be used for the purpose of explaining the small cosmological constant instead of the gauge hierarchy problem [6–9]. We will not discuss the matter further in this paper.

where the prime denotes the derivative with respect to  $y$ . Explicit forms of  $W$  are given in Section 3. In the following, we describe fluctuations. We concentrate our attention on analyses of the tensor modes of graviton, since the fluctuations decouple from the scalar and vector mode fluctuations and cannot be removed by any gauge choice [17].

The tensor mode fluctuations are taken as the line element (2.2) with the replacement  $\delta_{ij} \rightarrow \delta_{ij} + \kappa h_{ij}(x, y)$ . The fluctuations are decomposed over eigenfunctions:

$$h_{ij}(x, y) = \frac{1}{\sqrt{R}} \left[ h_{ij}^{(0)}(x) + \sum_{n=1}^{\infty} h_{ij}^{(n)}(x) \chi^{(n)}(y) \right], \quad (2.5)$$

where  $h_{ij}^{(0)}$  and  $h_{ij}^{(n)}$  stand for the zero mode and the  $n$ -th KK mode respectively. The eigenfunctions  $\chi^{(n)}$  are determined such that the four-dimensional spectra are governed by ordinary four-dimensional equations of motion. From the action (2.1) and the replaced line element, the kinetic action quadratic of tensor fluctuations are

$$S_{\text{kin}} = \int_0^{\pi R} \frac{dy}{R} A^2 \int d^4x \sqrt{-\tilde{g}} h_{ij}^{(0)} \square h_{ij}^{(0)} + \sum_{n=1}^{\infty} \int_0^{\pi R} \frac{dy}{R} A^2 \chi^{(n)2} \int d^4x \sqrt{-\tilde{g}} h_{ij}^{(n)} (\square - m_n^2) h_{ij}^{(n)}, \quad (2.6)$$

up to numerical factors. The determinant  $\tilde{g}$  and the d'Alembertian  $\square$  are made out of the four-dimensional metric  $d\tilde{s}^2 = -dt^2 + e^{2\sqrt{\lambda}t} \delta_{ij} dx^i dx^j$ . The zero mode is massless and the KK modes has the masses  $m_n$ . The normalization is canonically given by

$$\int_0^{\pi R} \frac{dy}{R} A^2 \chi^{(n)2} = 1. \quad (2.7)$$

The  $n$ -th eigenfunction  $\chi^{(n)}$  must satisfy the equation

$$\left( \partial_y^2 + 4 \frac{A'}{A} \partial_y + \frac{m_n^2}{A^2} \right) \chi^{(n)}(y) = 0, \quad (2.8)$$

which automatically leads to the orthogonality of the eigenfunctions. The boundary conditions are

$$\partial_y \chi^{(n)}(y) = 0, \quad \text{at } y = 0, \pi R. \quad (2.9)$$

It is seen that the  $n$ -th KK mass is derived from the equations (2.8) and (2.9), as follows. In the equation (2.8), the eigenfunction  $\chi^{(n)}$  is solved with two integration constants, one of which is an overall constant. In the solution, the other integration constant and the mass  $m_n$  are determined by the two boundary conditions (2.9). Thus the mass is completely described by the parameters of the model.

The interaction of the tensor fluctuations with matter is given by

$$S_{\text{int}} = \frac{\kappa}{\sqrt{R}} \int d^4x \sqrt{-g_4} h_{ij}^{(0)} T^{ij} + \sum_{n=1}^{\infty} \frac{\kappa}{\sqrt{R}} \chi^{(n)} \Big|_{y=\pi R} \int d^4x \sqrt{-g_4} h_{ij}^{(n)} T^{ij}, \quad (2.10)$$

with the four-dimensional energy momentum tensor  $T_{ij}$ . While the coupling for the zero mode is suppressed by  $\sqrt{R}/\kappa$  which is effectively of the order of Planck scale, the coupling for the KK modes may become large dependently on  $\chi^{(n)}$ . The size of  $\chi^{(n)}$  can be obtained after in the integral (2.7) the overall constant is fixed.

Here we comment on the bulk scalar field. In the equations (2.6)-(2.10), the scalar  $\Phi$  did not appear explicitly since the background scalar is the solution of the model and scalar fluctuations is dropped in the linearized analysis. The effects of the scalar field are included implicitly in the warp factor. The distinctive scalar function  $W$  would induce the distinctive warp factor. As shown in the following section, this brings a change to the masses and couplings.

### 3 Explicit analyses of the KK graviton masses and couplings

In this section, we explicitly perform analytic calculations of masses and couplings of KK graviton on the visible brane.

#### 3.1 Pure gravity

The first example is the pure gravity case

$$W = 6/L = \text{constant}, \quad (3.1)$$

where  $L$  denotes the five-dimensional curvature radius. In this case, the warp factor is given by [5]

$$A(y) = \cosh\left(\frac{y}{L}\right) - \sqrt{1 + \lambda L^2} \sinh\left(\frac{y}{L}\right), \quad (3.2)$$

with the orbifold condition imposed. It has been shown that the differential equation for the eigenfunctions  $\chi^{(n)}$  is transformed into a hypergeometric equation [10]. The normalization however has not been given in analytic methods. In fact it seems to be difficult to integrate hypergeometric functions in a finite interval. Since we have interest in the cosmological constant effects, we would like to advance analytic approach in some approximation using the smallness of the cosmological constant.

We define the dimensionless parameter

$$\epsilon \equiv \frac{\lambda L^2}{4} \ll 1, \quad (3.3)$$

and reexamine the KK mode fluctuations for the warp factor

$$A = e^{-y/L} \left[ 1 + \epsilon \left( 1 - e^{2y/L} \right) \right], \quad (3.4)$$

up to  $\mathcal{O}(\epsilon^2)$ <sup>§</sup>. Throughout this section all the equations will be described up to  $\mathcal{O}(\epsilon^2)$ . For simplicity of notation, we use the coordinate  $w = mL e^{y/L}$  and the indices of the modes will be

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<sup>§</sup>A similar approximation with respect to a small cosmological constant was used for studying issues of scalar potential, higher curvature and brane universe [9, 15, 16].

omitted hereafter as long as we do not mention. Then the eigenvalue equation (2.8) reduces to

$$\left[ \left\{ 1 + 2\epsilon \left( 1 - (w/w_0)^2 \right) \right\} \frac{\partial^2}{\partial w^2} - \left\{ 3 + 2\epsilon \left( 3 + (w/w_0)^2 \right) \right\} \frac{1}{w} \frac{\partial}{\partial w} + 1 \right] \chi = 0, \quad (3.5)$$

where  $w_0 = mL$  and we used  $\sqrt{\lambda}L e^{\pi R/L} \ll 1$ . We find the following solution to the equation (3.5):

$$\chi = w^2 \mathcal{J}_2 + \epsilon \left[ w^3 \mathcal{J}_3 + \frac{1}{3w_0^2} \left\{ 48w^3 \mathcal{J}_3 - 15w^4 \mathcal{J}_4 + w^5 \mathcal{J}_5 \right\} \right], \quad (3.6)$$

with

$$\mathcal{J}_n = c_1 J_n(w) + c_2 Y_n(w) \quad \text{for } n = 2, \dots, 5, \quad (3.7)$$

where  $c_1$  and  $c_2$  are integration constants. The solution with the first Bessel functions  $J_n$  is also represented in terms of Gauss' hypergeometric function as

$$\frac{1}{8} w^4 F \left( 2 - \frac{w_0}{2\sqrt{2\epsilon}}, 2 + \frac{w_0}{2\sqrt{2\epsilon}}, 3; \frac{2\epsilon}{1+2\epsilon} \left( \frac{w}{w_0} \right)^2 \right), \quad (3.8)$$

up to  $\mathcal{O}(\epsilon^2)$ .

From the solution (3.6), we calculate masses of KK modes according to the formulation in the previous section. After the substitution of the solution (3.6) into the boundary conditions (2.9) and a short calculation, it is found that the mass eigenvalue equation reduces to

$$J_1(w_1) + \frac{\epsilon}{3w_0^2} \left[ (12w_1 - w_1^3) J_2(w_1) + 9w_1^2 J_1(w_1) \right] \simeq 0, \quad (3.9)$$

where  $w_1 = mL e^{\pi R/L}$  and we used  $e^{\pi R/L} \gg 1$ . From this equation, we obtain the  $n$ -th KK mass

$$m_n = \frac{x_n}{L} e^{-\pi R/L} \left[ 1 - \frac{\epsilon}{3} \left( 1 - \frac{12}{x_n^2} \right) e^{2\pi R/L} \right], \quad (3.10)$$

where  $x_n$  indicates the  $n$ -th zero of the Bessel function  $J_1$ , for example,  $x_1 = 3.83$ ,  $x_2 = 7.02$ . In the equation (3.10), the first order term of  $\epsilon$  is multiplied by the factor  $e^{2\pi R/L}$  compared to the zero-th order term. The origin of this factor is  $e^{2y/L}$  in the warp factor (3.4). We speculate that if in some setting the warp factor receives larger contributions from the cosmological constant, the mass may be changed significantly. This possibility will be investigated in cases including bulk scalar fields. The analysis of the couplings can be carried out with the solution (3.6) and the mass (3.10). Using the effective Planck mass  $M_P^2 \simeq L/\kappa^2$ , we obtain the coupling of the first KK mode with matter

$$\frac{\kappa}{\sqrt{R}} \chi^{(1)}(\pi R) = \frac{\sqrt{2}}{M_P} e^{\pi R/L} \left[ 1 + \frac{4\epsilon}{3} \left( 1 - \frac{3}{x_1^2} \right) e^{2\pi R/L} \right], \quad (3.11)$$

which is of the order of  $\text{TeV}^{-1}$ . The factor  $e^{2\pi R/L}$  appears in similar to that of the mass. The effects of the cosmological constant on the mass and coupling is relevant equivalently to the effect on the warp factor. The derivation of the mass and coupling without requiring  $e^{\pi R/L} \gg 1$  will be given in Section 4.

### 3.2 Scalar-gravity

The second explicit model is the case with the scalar function

$$W(\Phi) = \frac{6}{L} - b\Phi^2 + \epsilon\delta W, \quad (3.12)$$

with the correction  $\delta W$  ( $< \epsilon^{-1}$ ) and the parameter  $b$  ( $> 0$ ). This form of the function  $W$  has been studied in Ref. [7]. In this case, the warp factor is obtained as<sup>¶</sup>

$$\ln A = -\frac{y}{L} + \epsilon \left[ e^{4y/L} - e^{2y/L} - \frac{1}{bL}(1 - e^{-2by}) \right]. \quad (3.13)$$

In order to make arguments clear, we focus on terms relevant near the boundaries. Then the warp factor can be approximated effectively as

$$A(y) \simeq e^{-y/L} \left[ 1 + \epsilon \left( e^{4y/L} - e^{2y/L} \right) \right]. \quad (3.14)$$

The factor  $e^{4y/L}$  is an effect induced by the mixing between the graviton and bulk scalar field. If other scalar functions are set, the scalar backgrounds may change  $\epsilon$  term in the warp factor. A more general warp factor and the corresponding eigenfunctions are given in Appendix A. In the warp factor (3.14), the solution for the eigenfunction is

$$\begin{aligned} \chi = w^2 \mathcal{J}_2 + \epsilon & \left[ \frac{1}{3w_0^2} \{ w^5 \mathcal{J}_5 - 15w^4 \mathcal{J}_4 + 48w^3 \mathcal{J}_3 \} \right. \\ & \left. + \frac{1}{5w_0^4} \{ w^7 \mathcal{J}_7 - 30w^6 \mathcal{J}_6 + 240w^5 \mathcal{J}_5 - 480w^4 \mathcal{J}_4 \} \right]. \end{aligned} \quad (3.15)$$

This solution is the equation (A.3) with the nonzero coefficients  $d_4 = 1$  and  $d_2 = -1$ . According to analyses in the previous subsection, we obtain the mass of the  $n$ -th KK mode,

$$m_n = \frac{x_n}{L} e^{-\pi R/L} \left[ 1 + \frac{\epsilon}{5} \left( 1 - \frac{24}{x_n^2} \right) e^{4\pi R/L} \right], \quad (3.16)$$

and the coupling,

$$\frac{\kappa}{\sqrt{R}} \chi^{(1)}(\pi R) = \frac{\sqrt{2}}{M_P} e^{\pi R/L} \left[ 1 - \frac{4\epsilon}{5} e^{4\pi R/L} \right]. \quad (3.17)$$

It is seen that the factor  $e^{4\pi R/L}$  occurs due to  $e^{4y/L}$  in the warp factor (3.14). Therefore in the two models where  $\epsilon$  expansion is applicable, the exponential enhancements are equivalent among the warp factor, masses and couplings. The enhanced contributions can become large dependently on the scalar function. In the following section, we examine how scalar potentials affect the KK graviton masses and couplings beyond  $\epsilon$  expansion.

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<sup>¶</sup>In Ref. [7], the solution for  $A$  was not shown explicitly.

## 4 Relation among the warp factor, the KK graviton masses and couplings

We would like to acquire a general observation about the relation among the effects of the cosmological constant on the warp factor, masses, and couplings. In order to distinguish quantities between the zero and nonzero cosmological constant cases, we introduce the following representations:

$$A[\epsilon], \quad V[\epsilon], \quad \chi[\epsilon], \quad m[\epsilon], \quad (4.1)$$

for the nonzero cosmological constant and

$$A[0], \quad V[0], \quad \chi[0], \quad m[0], \quad (4.2)$$

for the zero cosmological constant. In the equations (4.1) and (4.2), it is assumed that each corresponding quantity have the identical sign and that the warp factor  $A[0]$  maintains the order of cutoff energies on the two branes. The size of effects of cosmological constant is evaluated as the ratio

$$A_{\text{ratio}} = \frac{A[\epsilon] - A[0]}{A[0]}, \quad \chi_{\text{ratio}} = \frac{\chi[\epsilon] - \chi[0]}{\chi[0]}, \quad m_{\text{ratio}} = \frac{m[\epsilon] - m[0]}{m[0]}. \quad (4.3)$$

We first investigate how  $\chi_{\text{ratio}}$  is related to  $A_{\text{ratio}}$ . Note that the coupling is given by  $\kappa\chi(y = \pi R)/\sqrt{R}$ . The normalization (2.7) is

$$\int_0^{\pi R} \frac{dy}{R} A[\epsilon]^2 \chi[\epsilon]^2 = 1, \quad \text{for } \chi[\epsilon], \quad (4.4)$$

$$\int_0^{\pi R} \frac{dy}{R} A[0]^2 \chi[0]^2 = 1, \quad \text{for } \chi[0]. \quad (4.5)$$

Subtracting these equations each other reduces to

$$\chi_{\text{ratio}} = -\frac{A_{\text{ratio}}}{1 + A_{\text{ratio}}}. \quad (4.6)$$

This relation is satisfied independently of  $y$ . It is seen that the coupling of KK mode with matter are determined only by  $A_{\text{ratio}}$ . When  $A_{\text{ratio}}(y = \pi R)$  is negative, the coupling necessarily increases. However, the coupling of the same size would be obtained even in the zero cosmological constant case if a distinctive compactification radius is chosen. One must take the mass into account in order to identify the nonzero cosmological constant effects, as follows.

We evaluate the ratio  $m_{\text{ratio}}$  by examining the equation (2.8),

$$\left[ \partial_y^2 - 4 \frac{A[\epsilon]'}{A[\epsilon]} \partial_y + \frac{m[\epsilon]^2}{A[\epsilon]^2} \right] \chi[\epsilon] = 0, \quad \text{for } \chi[\epsilon], \quad (4.7)$$

$$\left[ \partial_y^2 - 4 \frac{A[0]'}{A[0]} \partial_y + \frac{m[0]^2}{A[0]^2} \right] \chi[0] = 0, \quad \text{for } \chi[0]. \quad (4.8)$$



From the equations above, the equation (4.7) is written as

$$\frac{m[\epsilon]^2}{A[\epsilon]^2} - \frac{m[0]^2}{A[0]^2} + \left[ 4 \frac{A[0]'}{A[0]} - 6 \frac{\chi[0]'}{\chi[0]} + 6 \frac{A'_{\text{ratio}}}{1 + A_{\text{ratio}}} \right] \frac{A'_{\text{ratio}}}{1 + A_{\text{ratio}}} - \frac{A''_{\text{ratio}}}{1 + A_{\text{ratio}}} = 0. \quad (4.9)$$

From the boundary conditions (2.9) and the equation (4.6), this equation at the boundaries becomes

$$\frac{m[\epsilon]^2 - 4\epsilon/(3L^2)}{A[\epsilon]^2} - \frac{m[0]^2}{A[0]^2} + \frac{2}{3}(V[\epsilon] - V[0]) = 0 \quad (4.10)$$

where  $V$  is defined by the equation (2.3). Then we obtain

$$m_{\text{ratio}} = -1 + (1 + A_{\text{ratio}}) \sqrt{1 - \frac{2A[0]^2}{3m[0]^2}(V[\epsilon] - V[0]) + \frac{4\epsilon}{3m[0]^2 L^2 (1 + A_{\text{ratio}})^2}}, \quad (4.11)$$

which is derived without the simplification  $e^{\pi R/L} \gg 1$ . From the assumption for  $A[0]$  in the beginning of this section, the last term in the square root is negligible and  $A[0]/m[0] \sim L$ . The mass depends on the potential difference ( $V[\epsilon] - V[0]$ ) as well as  $A_{\text{ratio}}$ . Thus the effect of the cosmological constant is different from a simple choice of the compactification radius.

From the equations (4.6) and (4.11), the coupling and mass become of order  $\mathcal{O}(10)\%$  larger than those of the zero cosmological constant case if the warp factor difference ratio  $A_{\text{ratio}}$  or the scalar potential difference ratio  $(V[\epsilon] - V[0])L^2$  are a negative value of order  $\mathcal{O}(10)\%$ . This provides a phenomenological possibility of cosmological constant effects at TeV scale.

## 5 Summary and discussions

We have calculated masses and interactions of KK graviton in models with a warped extra dimension and a four-dimensional cosmological constant. In the pure gravity case, we have reexamined the eigenfunction by expanding the warp factor into power series of the dimensionless cosmological constant  $\epsilon$ . Correspondingly to the exponential factor  $e^{2y/L}$  in the  $\epsilon$  term of the warp factor, it has been found that the mass and coupling have the factor  $e^{2\pi R/L}$  in  $\epsilon$  term. In the case with the scalar potential given in Ref. [7], we have found that the mass and coupling have the factor  $e^{4\pi R/L}$  in the  $\epsilon$  term correspondingly to the warp factor  $e^{4y/L}$ . Thus it has been shown that the bulk scalar field increases the cosmological constant effects. In addition, the effects of the cosmological constant on the warp factor are equivalent to those of the masses and couplings. Beyond the power expansion, we have also found generally the masses and couplings which are described in terms of the warp factors and scalar potentials. From this indication, we have presented a possibility of phenomenological effects of the cosmological constant.

Our solutions for the graviton eigenfunctions may be useful for studying the holographic principle [18, 19]. The original Randall-Sundrum model is expected to be included in this context [20]. It is likely that quantum theories on branes have geometrical settings in higher-dimensional spacetime. For quantum theories with a cosmological constant, it was found that

there is an ambiguity of the vacuum [21]. In fact it has been shown that this seems to be an artifact which can be evaded [22]. It is worth while to seriously study quantum theories for the nonzero cosmological constant case and to proceed on a clear understanding of the geometrical settings [23, 24]. Since our solutions for the eigenfunctions make the cosmological constant effects manifest, the analyses with the solutions would provide direct deviations from the zero cosmological constant case.

Finally we would like to mention scalar potentials. We have not asked what form of scalar potential induces phenomenologically significant cosmological constant effects. It is interest to exhaustively investigate this question using ansatz for scalar function. If a viable scalar function is found, it is also interesting to examine how the models can be embedded in fundamental theory such as string theory following proposals in Ref. [25].

## Acknowledgments

The authors thank K. Ghoroku for discussions. This work is supported by the grant-in-aid for scientific research on priority areas: “Progress in elementary particle physics of the 21st century through discoveries of Higgs boson and supersymmetry” (No. 441).

## A Eigenfunctions in $\epsilon$ expansion

In this appendix, we present eigenfunctions in the case where the warp factor has the following general form of  $\epsilon$  expansion:

$$A = e^{-y/L} \left[ 1 + \epsilon \sum_{n=0}^p d_{2n} e^{2ny/L} \right], \quad (\text{A.1})$$

where the coefficients  $d_{2n}$  are order of  $\mathcal{O}(1)$ . For graviton, the equation (2.8) is assumed,

$$\left( \partial_y^2 + 4 \frac{A'}{A} \partial_y + \frac{m^2}{A^2} \right) \chi(y) = 0. \quad (\text{A.2})$$

Then we find the following solution:

$$\begin{aligned} \chi = & w^2 \mathcal{J}_2 + \epsilon \left[ -d_0 w^3 \mathcal{J}_1 - \frac{2}{3} \frac{d_2}{w_0^2} w^5 \mathcal{J}_1 + \sum_{n=0}^1 \frac{q_{n,1}}{w_0^{2n}} \mathcal{I}_n \right. \\ & \left. + \sum_{l=2}^p \left( \frac{-2}{l+2} \frac{d_{2l}}{w_0^{2l}} w^{2l+3} \mathcal{J}_1 + \sum_{n=2}^l \frac{q_{n,l}}{w_0^{2n}} \mathcal{I}_n \right) \right], \end{aligned} \quad (\text{A.3})$$

with

$$\mathcal{I}_n = (n+2) w^{2n+2} \mathcal{J}_2 - w^{2n+3} \mathcal{J}_1, \quad (\text{A.4})$$

$$q_{0,1} = -2d_2, \quad (\text{A.5})$$

$$q_{1,1} = -\frac{1}{3} d_2, \quad (\text{A.6})$$

$$q_{l,l} = -\frac{3l}{(l+2)(l+1)} d_{2l}, \quad \text{for } l \geq 2, \quad (\text{A.7})$$

$$q_{n,l} = (-2)^{l-n} \frac{l+1}{n+1} \left[ \prod_{k=n+1}^l \frac{k(k-2)(k+2)}{k+1} \right] q_{l,l}, \quad \text{for } l > n \geq 2. \quad (\text{A.8})$$

Instead of the equation (A.2), we can assume the following eigenfunction equation for bulk gauge boson,  $\chi^{(A)}$ :

$$\left( \partial_y^2 + 2 \frac{A'}{A} \partial_y + \frac{m^2}{A^2} \right) \chi^{(A)}(y) = 0. \quad (\text{A.9})$$

Then we obtain the following solution:

$$\begin{aligned} \chi^{(A)}(y) = & w \mathcal{J}_1 + \epsilon \left[ d_0 (w \mathcal{J}_1 - w^2 \mathcal{J}_0) \right. \\ & \left. + \sum_{l=1}^p \left( \frac{-1}{l+1} \frac{d_{2l}}{w_0^{2l}} w^{2l+2} \mathcal{J}_0 + \sum_{n=1}^l \frac{q_{n,l}^{(A)}}{w_0^{2n}} \mathcal{I}_n^{(A)} \right) \right], \end{aligned} \quad (\text{A.10})$$

with

$$\mathcal{I}_n^{(A)} = (n+1)w^{2n+1}\mathcal{J}_1 - w^{2n+2}\mathcal{J}_0, \quad (\text{A.11})$$

$$q_{l,l}^{(A)} = -\frac{l}{(2l+1)(l+1)}d_{2l}, \quad (\text{A.12})$$

$$q_{n,l}^{(A)} = (-2)^{l-n}\frac{2l+1}{2n+1}\left[\prod_{k=n+1}^l\frac{k(k-1)(k+1)}{2k+1}\right]q_{l,l}^{(A)}, \quad \text{for } l > n \geq 1. \quad (\text{A.13})$$

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